The given point lies on the terminal side of an angle \( \theta \) in standard position. Find the values of the six trigonometric functions of \( \theta \).

7. \((-8, 15)\)

**SOLUTION:**

Use the values of \( x \) and \( y \) to find \( r \).

\[
r = \sqrt{x^2 + y^2} = \sqrt{(-8)^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17
\]

Use \( x = -8, y = 15, \) and \( r = 17 \) to write the six trigonometric ratios.

\[
\begin{align*}
\sin \theta &= \frac{y}{r} = \frac{15}{17} \\
\cos \theta &= \frac{x}{r} = \frac{-8}{17} \\
\tan \theta &= \frac{y}{x} = \frac{-15}{8} \\
\csc \theta &= \frac{r}{y} = \frac{17}{15} \\
\sec \theta &= \frac{r}{x} = \frac{-17}{8} \\
\cot \theta &= \frac{x}{y} = \frac{-8}{15}
\end{align*}
\]

Sketch each angle. Then find its reference angle.

20. \(\frac{11\pi}{3}\)

**SOLUTION:**

A coterminal angle is \(\frac{11\pi}{3} - 2\pi = \frac{5\pi}{3}\), which lies in Quadrant IV. So, the reference angle is \(\theta' = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}\).

21. \(-405^\circ\)

**SOLUTION:**

A coterminal angle is \(-405^\circ + 360^\circ(2) = 315^\circ\). The terminal side of \(315^\circ\) lies in Quadrant IV, so its reference angle is \(360^\circ - 315^\circ = 45^\circ\).

22. \(-75^\circ\)

**SOLUTION:**

A coterminal angle is \(-75^\circ + 360^\circ = 285^\circ\). The terminal side of \(285^\circ\) lies in Quadrant IV, so its reference angle is \(360^\circ - 285^\circ = 75^\circ\).

24. \(\frac{13\pi}{6}\)

**SOLUTION:**

A coterminal angle is \(\frac{13\pi}{6} + 2(-1)\pi = \frac{\pi}{6}\). The terminal side of \(\frac{\pi}{6}\) lies in Quadrant I, so the reference angle is \(\frac{\pi}{6}\).
Find the exact values of the five remaining trigonometric functions of \( \theta \).

33. \( \tan \theta = 2 \), where \( \sin \theta > 0 \) and \( \cos \theta > 0 \)

**SOLUTION:**
To find the other function values, you must find the coordinates of a point on the terminal side of \( \theta \). You know that \( \sin \theta \) and \( \cos \theta \) are positive, so \( \theta \) must lie in Quadrant I. This means that both \( x \) and \( y \) are positive.

Because \( \tan \theta = \frac{y}{x} \text{ or } \frac{2}{1} \), use the point (1, 2) to find \( r \).

\[
r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 2^2} = \sqrt{5}
\]

Use \( x = 1 \), \( y = 2 \), and \( r = \sqrt{5} \) to write the five remaining trigonometric ratios.

\[
\sin \theta = \frac{y}{r} = \frac{2}{\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5} \quad \csc \theta = \frac{r}{y} = \frac{\sqrt{5}}{2}
\]

\[
\cos \theta = \frac{x}{r} = \frac{1}{\sqrt{5}} \text{ or } \frac{\sqrt{5}}{5} \quad \sec \theta = \frac{r}{x} = \frac{\sqrt{5}}{1} \text{ or } 1
\]

\[
\cot \theta = \frac{x}{y} \text{ or } \frac{1}{2}
\]